**PROPORTION AND PERCENT OVERVIEW**

Before you start, you should understand fractions and decimals.

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**Ratios Compare Two Numbers.**

For example, if an animal shelter has 6 cats and 7 dogs, the ratio of cats to dogs in the animal shelter is 6 to 7.

This ratio can be expressed in several ways.

<table>
<thead>
<tr>
<th>( \frac{6}{7} )</th>
<th>( 6 : 7 )</th>
<th>( \frac{6}{7} )</th>
</tr>
</thead>
</table>

Ratios can be expressed in several forms; we strongly urge students to write ratios in the form that looks like a fraction. This is a familiar form that will make it easier for the student to perform calculations:

\[
\frac{6}{7}
\]

We also strongly urge students to label the ratios when they write them.

\[
\text{cats} \quad \frac{6}{7} \quad \text{dogs}
\]

**WRITE THE RATIO IN THE ORDER THAT IT IS WRITTEN IN THE PROBLEM.**
The order of the ratio depends on its presentation. The number mentioned first is written first. Notice that the dogs come before the cats on this page. We, therefore, write the ratio with dogs coming first. If an animal shelter has 7 dogs and 6 cats, the ratio of dogs to cats in the animal shelter is 7 to 6. Although there is no difference in the actual number of dogs and cats on this page, we write the ratio differently.

\[
\frac{7}{6} \quad 7:6 \quad \frac{7}{6}
\]

This ratio is now in its simplest form and cannot be further simplified. Although it looks like an improper fraction, it isn’t.

**ALTHOUGH A RATIO MAY LOOK LIKE A FRACTION AND HAS SOME CHARACTERISTICS OF A FRACTION, DO NOT THINK OF IT AS A FRACTION.**

In the following problem, the ratio must be simplified.

Yesterday I took a bunch of coins out of my pocket. There were fifteen dimes and six nickels. What is the ratio of dimes to nickels?

Note that in the problem, there are fifteen dimes and 6 nickels or 15 to 6. Since dimes are mentioned first, the ratio is expressed: dimes to nickels.

\[
\frac{15}{6}
\]

As with a fraction, we do not leave the ratio in this form. Again as with fractions, we find the highest common divisor. In this case both 15 and 6 can be divided by 3.
When we regroup the coins, we can easily see that for every five dimes, there are 2 nickels. We write the ratio in its simplest form. Do not forget to label the ratios.

\[
\begin{array}{ccc}
\text{dimes} & 15 & 15 \div 3 = 5 \\
\text{nickels} & 6 & 6 \div 3 = 2 \\
\end{array}
\]

\[
\begin{array}{ccc}
\text{dimes} & 5 & \\
\text{nickels} & 2 & \\
\end{array}
\]

A Proportion Compares Two Ratios

From the previous problem we discovered that 15:6 is equivalent to 5:2. A proportion is an equation that compares two ratios.

\[
\begin{array}{ccc}
\text{dimes} & 15 & 5 \\
\text{nickels} & 6 & 2 \\
\end{array}
\]

In another example, there is an office with ten people. There are two women for every three men. That is, the ratio of women to men is 2:3. In this office there are actually 4 women and 6 men. This can be written as a proportion.

\[
\begin{array}{ccc}
\text{women} & 4 & 2 \\
\text{men} & 6 & 3 \\
\end{array}
\]

Notice that by cross multiplying:

\[
\begin{array}{ccc}
4 \times 3 &= 12 \\
6 \times 2 &= 12 \\
\end{array}
\]
How to Solve Simple Proportions

We use the attribute of proportions that the products of cross multiplication are equal. Notice the example on the previous page:

\[
\frac{4}{6} = \frac{2}{3} \quad \text{Notice that by cross multiplying:} \quad \frac{4 \times 3}{6 \times 2} = \frac{12}{12}
\]

We can use this attribute to solve simple proportions. Remember that division is the opposite of multiplication. Note the relationship: \(4 \times 3 = 12\) \(\div\) \(3 = 4\)

\[
\frac{m}{6} = \frac{2}{3} \quad \text{In this example, we want to find the value of } m.
\]
1. Cross multiply: \(6 \times 2 = 12\)
2. Divide: \(12 \div 3 = 4\)
3. Therefore \(m = 4\)

Let us try some more examples.

\[
\frac{3}{5} = \frac{d}{45} \quad \text{In this example, we want to find the value of } d.
\]
1. Cross multiply: \(3 \times 45 = 135\)
2. Divide: \(135 \div 5 = 27\)
3. Therefore \(d = 27\)

\[
\frac{3}{c} = \frac{12}{16} \quad \text{In this example, we want to find the value of } c.
\]
1. Cross multiply: \(3 \times 16 = 48\)
2. Divide: \(48 \div 12 = 4\)
3. Therefore \(c = 4\)

\[
\frac{1}{2} = \frac{f}{30} \quad \text{In this example, we want to find the value of } f.
\]
1. Cross multiply: \(1/2 \times 30 = 15\)
2. Divide: \(15 \div 3 = 5\)
3. Therefore \(f = 5\)

PRACTICE USING MATEMATIKO
Proportion Word Problem Example:

Harry has a recipe that calls for 2 eggs to make 16 cranberry-apricot bars. He plans to bake 24 bars for his son’s fifth grade class. How many eggs will he need to have?

\[
\frac{\text{eggs}}{\text{bars}} = \frac{2}{16} = \frac{X}{24}
\]

Cross Multiply: \[2 \times 24 = 48\]
Divide: \[48 \div 16 = 3\]

Examine the groupings of the eggs and cranberry apple bars above. Note that for every 8 bars, there is 1 egg. Also note that 16 bars correspond to 2 eggs (information given in problem), and that 24 bars correspond to three eggs. Also note that if you simplify 2 to 16, you get 1:8.

A Percent is a Ratio

A percent is a ratio. It compares a number to 100. We use the word **cent**. Cent is the shortened form of the Latin word **centum** which means 100.

For example we all know that a dollar is equal to 100 cents. If you have 14 cents in your pocket, you have 14 out of 100 cents or 14% of a dollar.

This can be written as a ratio:
\[
\frac{\text{part}}{\text{whole}} = \frac{14}{100}
\]

It can also be written in the familiar decimal form:
\[.14\text{ or } \$ .14\]

The line in a percent ratio can be thought of as the word **per**. Therefore 14% in the fraction form can be read as 14 per 100 or 14 per cent.

You can also think of the percent bar as the word of. You can refer to 14% as 14 out of 100 or just 14 of 100.
USE PROPORTIONS TO SOLVE PERCENT PROBLEMS

My son’s class has 16 children. There are 4 boys and 12 girls in the class. Therefore the ratio of boys to the whole class is 4:16. Using the information about the distribution of boys and girls in my son’s class we can come up with three types of percent problems.

Before discussing the three types of percent problems that can be derived from the above information, we will show that the ratio of boys to the whole class (4:16) can be written as 25%.

As you know, a proportion is an equation that compares two ratios. Notice the following proportion. The ratio on the right refers to the number of boys (part) compared to the (whole class). The ratio on the left is the percent. See label left of the proportion.

This proportion can be thought of as: Four of the sixteen students in the class are boys. Therefore 25 percent of the class is boys.

\[
\frac{\text{PART}}{\text{WHOLE}} = \frac{25}{100} = \frac{4}{16}
\]

The validity of the proportion can be shown by cross-multiplying.

\[
4 \times 100 = 400 \\
16 \times 25 = 400
\]

We will use this cross multiplication attribute of proportions to solve three types of percent problems.

- Find the percent when given the whole and the part.
- Find the part when given the whole and the percent
- Find the whole given the percent and the part.
THREE TYPES OF PERCENT PROBLEMS
FIND A PERCENT- GIVEN WHOLE AND PART

My son’s class has 16 children. There are 4 boys and 12 girls in the class. What percent of the children in my son’s class are boys?

To find the percent of boys in the class, use a proportion. You are asking yourself to compare the relation of 4:16 to some number (WHAT) compared to 100. The $X$ in the equation below refers to some unknown number (percent). Hint: when you see the question: what percent, you always write $X/100$ on the left side of the equation.

\[
\frac{\text{part}}{\text{whole}} = \frac{X}{100} = \frac{4}{16}
\]

Calculation

\[
4 \times 100 = 400
\]

\[
\frac{25}{1600}
\]

Therefore 25% of the pupils in my son’s class are boys.

The ratio of girls to the whole class is 12/16.

\[
\frac{\text{part}}{\text{whole}} = \frac{X}{100} = \frac{12}{16}
\]

\[
12 \times 100 = 1200
\]

\[
\frac{75}{1600}
\]

If you figured that 75% of the pupils in my son’s class are girls, you are correct. Note that if the boys comprise 25% of the class and the girls comprise 75% of the class, the total (whole) class is 100%.

PRACTICE USING MATEMATIKO
FIND THE PART- GIVEN THE PERCENT AND WHOLE

We will use the same class that we used in the previous example. This time you will be given the percent and will be asked to find the part.

My son’s class has 16 children. 25% of the class is made up of boys. How many boys are in the class?

In this example the unknown number, the answer we are looking for is the part. Again we will use the letter \( X \) to represent the unknown quantity or the answer. Hint (1): Since you are given 25%, you write \( \frac{25}{100} \) on the left side. Hint (2): Since the word class refers to 16 children and the word class comes after the word “of,” 16 refers to the whole class and is placed under the bar on the right side.

\[
\frac{\text{part}}{\text{whole}} = \frac{25}{100} = \frac{X}{16}
\]

\[
16 \times 25 = \frac{400}{100} = 4
\]

Therefore there are four boys in my son’s class.

PRACTICE USING MATEMATIKO

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FIND THE WHOLE- GIVEN THE PERCENT AND PART

My son’s class has 4 boys. The boys make up 25% of the class. How many children are in the class altogether?

In this case, we know that there are four boys in the class and the boys make up 25% or 25/100 of the class. We are now looking for the whole. Hint (1): Since you are given 25%, you write 25/100 on the left side. Hint (2): Since the word ‘class’ comes after the word “of,” and we are looking for the number of students in the class(whole), the X is placed under the bar on the right side.

\[
\frac{\text{part}}{\text{whole}} = \frac{25}{100} = \frac{4}{X}
\]

\[
4 \times 100 = 400 \div 25 = 16
\]

There are 16 children in my son’s class.

PRACTICE USING MATEMATIKO

Now that you understand how to solve three types of percent documents separately, now try a set of mixed examples.

PRACTICE USING MATEMATIKO
SOLVING PERCENT PROBLEMS USING SHORTCUTS
Study this page if you want to use a short cut.

Instead of solving some percent problems with a proportion, a percent problem can be solved by simple division.

<table>
<thead>
<tr>
<th>PERCENT</th>
<th>DIVIDE BY</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 %</td>
<td>20</td>
</tr>
<tr>
<td>10 %</td>
<td>10</td>
</tr>
<tr>
<td>12 ½ %</td>
<td>8</td>
</tr>
<tr>
<td>20 %</td>
<td>5</td>
</tr>
<tr>
<td>25 %</td>
<td>4</td>
</tr>
<tr>
<td>33 1/3 %</td>
<td>3</td>
</tr>
<tr>
<td>50 %</td>
<td>2</td>
</tr>
</tbody>
</table>

FIND THE PART- GIVEN THE PERCENT AND WHOLE
Remember the problem on page 8 when we had to calculate 25 % of 16. We used the following proportion to find the part to find the answer is 4.

\[
\frac{\text{part}}{\text{whole}} = \frac{25}{100} = \frac{X}{16}
\]

Using the shortcut method, by looking at the chart above for 25 %, we see that 25 percent of a number is the same as \(\frac{1}{4}\) of a number. We divide 16 by 4.

\[
16 ÷ 4 = 4
\]

Therefore 25 % of 16 is 4.

Another example
Find 50% of 150.

Look above at chart. It says to find 50% of a number. We divide 150 by 2.

\[
150 ÷ 2 = 75
\]

Therefore 50% of 150 is 75.